1 Probability Distributions

1.1 Concepts

1.

Distribution	PMF	Example
Uniform	If $\#R(X) = n$, then $f(x) = \frac{1}{n}$ for all $x \in R(X)$.	Dice roll, $f(1) = f(2) = \cdots = f(6) = \frac{1}{6}$.
Bernoulli Trial	$f(0) = 1 - p$, $f(1) = p$	Flipping a biased coin
Binomial	$f(k) = \binom{n}{k} p^k (1-p)^{n-k}$	p is probability of success. Repeat n Bernoulli trials. Number of 6's rolled when rolling 10 die is $f(k) = \binom{10}{k}(1/6)^k(5/6)^{10-k}$.
Geometric	$f(k) = (1-p)^k p$	k failures and then a success.
Hyper-Geometric	$f(k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}}$	Counting the number of red balls I pick out of n balls drawn if there are m red balls out of N balls total.
Poisson	$f(k) = \frac{\lambda^k e^{-\lambda}}{k!}$	Count the number of babies born today if on average there are 3 babies born a day.

1.2 Examples

2. I am picking cards out of a deck. What is the probability that I pull out 1 heart out of 5 cards if I pull with replacement? If I pull 5 cards at once?

3. What is the probability that first heart is the third card I draw (with replacement)?

1.3 Problems

4. True   False   We cannot talk about Bernoulli trials for rolling a 5 because there are 6 outputs and we need 2 for a Bernoulli trial.

5. True   False   The geometric distribution, like the hyper-geometric distribution, assumes that the trials are dependent (without replacement).

6. In a class of 50 males and 80 females, I give out 3 awards randomly. What is the probability that females will win all 3 awards if the awards must go to different people? What about if the same person can win all three awards?

7. At Berkeley, there is an equal number of people aged $18, 19, \ldots, 27$. I cold call someone at random and ask for their age. What is the PMF for their age? Suppose that undergraduates are aged 18 though 21 inclusive. What is the probability that I have to call 10 people until I call an undergraduate (the undergraduate is the 10th person I call)? What is the probability that I call 4 undergraduates out of 10 people I call (if I can call someone more than once)?

8. For a lottery, 6 distinct numbers are drawn out of 60 and to win, you need to match all 6 numbers. What is the probability that I win? If I buy 100 different tickets, what is the probability that I win?

1.4   Extra Problems

9. In a class of 80 males and 60 females, I give out 3 awards randomly. What is the probability that 2 females will win awards if the awards must go to different people? What about if the same person can win all three awards?

10. At Berkeley, there is an equal number of people aged $18, 19, \ldots, 27$. I cold call someone at random and ask for their age. What is the PMF for their age? Suppose that undergraduates are aged 18 though 21 inclusive. What is the probability that I have to call 30 people until I call an undergraduate (the undergraduate is the 30th person I call)? What is the probability that I call 8 undergraduates out of 15 people I call (if I can call someone more than once)?

11. For a lottery, 4 distinct numbers are drawn out of 40 and to win, you need to match all 4 numbers. What is the probability that I win? If I buy 30 different tickets, what is the probability that I win?